

IONORT: a Windows software tool to calculate the HF ray tracing in the ionosphere

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Abstract

This paper describes an applicative software tool, named IONORT (IONOspheric Ray Tracing), for calculating a three-dimensional ray tracing of high frequency waves in the ionospheric medium. This tool runs under Windows operating systems and its friendly graphical user interface facilitates both the numerical data input/output and the two/three-dimensional visualization of the ray path. In order to calculate the coordinates of the ray and the three components of the wave vector along the path as dependent variables, the core of the program solves a system of six first order differential equations, the group path being the independent variable of integration. IONORT uses a three-dimensional electron density specification of the ionosphere, as well as by geomagnetic field and neutral particles-electrons collision frequency models having validity in the area of interest.

Keywords: ray tracing, ray path, ionospheric models.

25 **1. Introduction**

26 Ray tracing (RT) is a numerical technique used to determine the path of a high frequency (HF)
27 radio wave in anisotropic and inhomogeneous media different from the vacuum (Budden, 1988).
28 The technique works properly if the refractive index is assumed to be known in each point of the
29 considered region. In the limits of the ray theory it is possible to approximate the wavelength to
30 zero, simplifying consequently the differential equations describing the propagation of the wave in a
31 suitable way, i.e. the ray path. Hence, three-dimensional (3-D) RT algorithms calculate the
32 coordinates reached by the wave vector and its three components, the group time delay of the wave
33 along the path and other optional quantities (geometrical and phase path, absorption, polarization,
34 etc.). In order to accomplish these tasks, the RT programs integrate at least six differential
35 equations, plus other equations when additional quantities, like for instance Doppler frequency
36 shift, are required. These kinds of algorithms were developed in the 1950's (Haselgrove, 1955;
37 Duziak, 1961; Croft and Gregory, 1963) for old mainframes that were able to give only a numerical
38 output. Nowadays, these programs have been optimized and adapted to Over The Horizon radar
39 applications (Coleman, 1998; Nickish, 2008) by using powerful computers and devices for a real-
40 time use.

41 This paper deals with a software tool, named IONORT, whose RT algorithm is based on a system
42 of first order differential equations with Hamiltonian formalism that are solved for a geocentric
43 spherical coordinate system. The corresponding software (that can be downloaded at the site
44 ftp://ftp.ingv.it/pub/adriano.azzarone/ionort_0.7.2.zip) is written in MATLAB for the input and the
45 output routines, while the integration algorithm is derived from the one that was coded in Fortran by
46 Jones and Stephenson (1974). In the next future, a whole package coded in MATLAB is planned.
47 The ionosphere considered by this software tool is represented by 3-D ionospheric regional models
48 elaborated at the Istituto Nazionale di Geofisica e Vulcanologia. An analytical standard Chapman
49 modeled ionosphere (Chapman, 1931) useful mainly for test purpose complete the whole package.

50

2. Generality on the ionospheric ray tracing algorithm

Ray tracing techniques rely on a comprehensive specification of the ionosphere in terms of electron density, neutral particles-electrons collision frequency, and geomagnetic field. As an example, Fig. 1 shows a 3-D matrix of the electron density where in each cell C_{ijk} (here, the indexes i, j , and k are respectively the longitude, the latitude, and the altitude and these depend on the corresponding matrix resolution) the electron density of the ionospheric medium, and hence the corresponding complex refractive index n , has a defined value.

Fig. 1 shows also a possible ray path from a transmitting (TX) point to a receiving (RX) point across the cells of the 3-D electron density matrix with the frequency and direction of the wave, and the position of the TX point given as input. The RT algorithm integrates the following partial differential equations

$$\frac{dr_i}{d\tau} = \frac{\partial H(r_i, k_i)}{\partial k_i} \quad (1.1)$$

$$\frac{dk_i}{d\tau} = -\frac{\partial H(r_i, k_i)}{\partial r_i} \quad (1.2)$$

where $i=1, \dots, 4$, $H(r_i, k_i)$ is the Hamiltonian, r_i , and k_i , are respectively the generalized coordinates and momenta, while the independent variable τ must be a monotonic increasing quantity (represented in our case by the group path) (Weinberg, 1962). (1.1) and (1.2) are solved for a geocentric spherical coordinate system (r, θ, φ) , and according to the wave vector components, as shown in Fig. 2 (Bianchi and Bianchi, 2009; Bianchi et al., 2010). The essential differential equations that are integrated are given more explicitly in the Appendix A.

75 **3. IONORT: description of the program**

76 IONORT is structured in three main blocks:

77

78 a) INPUT GRAPHICAL USER INTERFACE;

79 b) INTEGRATION ALGORITHM;

80 c) OUTPUT GRAPHICAL USER INTERFACE.

81

82 Fig. 3 shows the flowchart of the IONORT application.

83 The block a), developed in MATLAB, firstly reads a file named “DATA_default.ini” to initialize
84 the default inputs related to all the computational parameters needed by the ray-tracing algorithm.
85 After this phase of initialization, it visualizes a graphical user interface (GUI, see Fig. 4 or Fig. 5)
86 by which the user can modify the default inputs, and then it generates a file “DATA_in.txt”
87 representing the user input for the integration executable code, written in Fortran, that is the block
88 b). “DATA_in.txt” is then nothing but a copy of the file “DATA_default.ini” modified according to
89 the choices made by the user. “DATA_in.txt” is then the actual input of the Fortran core, which
90 reads it as a vector W of 400 components. Table 1 shows the first 25 components of such a vector,
91 in particular: the geographical coordinates of the TX point, the height (in km) of the TX and the RX
92 points at which the program must start and stop respectively, the azimuth and elevation angles (in
93 degrees), the wave operating frequency (in MHz), the polarization of the ray (ordinary or
94 extraordinary), the geographical coordinates of the geomagnetic pole, the number of hops, and some
95 needed constants like for instance the Earth radius. The user can modify some of these input
96 parameters, as well as the analytical or numerical models representing the ionosphere (according to
97 what is shown by Table 2), by filling and checking the corresponding boxes of the “Main
98 parameters”, “Step”, “Model”, and “Ray” frames of the GUI. Once the parameters have been set,
99 the “RUN” button launches the integration algorithm.

100 The block b), which is the core of the application, is represented by this integration algorithm that
101 is coded as a Fortran executable. In order to integrate step by step the differential equations
102 illustrated in the Appendix A, this executable performs all the computational operations using either
103 the 4-order Runge-Kutta (RK) method (Press et al., 1996) or the Adams-Bushford predictor and the
104 Adams-Moulton corrector methods (ABAM) (Press et al., 1996). Using these, the ray path of the
105 wave in spherical coordinates is calculated.

106 Once this task ended, the block c), besides saving the numerical output in a file “RToutput.txt”,
107 visualizes the results in the GUI where also 2-D and 3-D graphical elaborations of the ray path are
108 performed. The 2-D visualization is plotted at the bottom of the GUI in a plane section having
109 constant azimuth. The 3-D visualization is plotted on the right side of the GUI. The numerical
110 output of some relevant parameters like the latitude and the longitude of the arrival point, the
111 ground range distance on the Earth’s surface, the maximum altitude of the path trajectory (apogee),
112 and the time delay of the ray along the whole path (group delay) are shown in the “Results” frame
113 of the GUI.

114 IONORT can run both with a fixed operating frequency and with a frequency-step procedure. The
115 same is for the elevation and the azimuth angles. Fig. 4 and Fig. 5 show two examples of
116 elaboration, by taking into account a TX point at 43.06°N of latitude and at 10.03°E of longitude,
117 the former for a fixed frequency equal to 6 MHz and for a 5° elevation-step procedure from 0° to
118 30° , the latter for a fixed elevation angle equal to 15° and for a 2 MHz frequency-step procedure
119 from 2 MHz to 24 MHz.

120

121 **4. Description of the integration computational code**

122 The main task of IONORT is the integration of the first-order differential equation system given in
123 the Appendix A. The discrete form of the system can be write as

124

$$\frac{d y_i}{d \tau} = f_i(\tau, y_1, \dots, y_N), \quad (2)$$

where $i=1, \dots, N$, y_i are the dependent variables (coordinates and wave vector components) and τ is the independent variable. For each of the six equations of the system, at the step $n+1$ the classical 4-order RK formula gives

$$y_{n+1} = y_n + h \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4), \quad (3.1)$$

with

$$\tau_{n+1} = \tau_n + h, \quad (3.2)$$

where n and h represent respectively the integration step number and the integration step value, $k_1=f(\tau_n, y_n)$, $k_2=f(\tau_n+h/2, y_n+hk_1/2)$, $k_3=f(\tau_n+h/2, y_n+hk_2/2)$, and $k_4=f(\tau_n+h, y_n+hk_3)$. Alternatively, the system can be solved using the ABAM method. In the RK method the independent variable τ can assume values from tens of meters to a few kilometers, while in the ABAM method there is an adaptive step according with a maximum and a minimum tolerated error.

With regard to the refractive index calculation routine, as we have already mentioned in the paragraph 3, besides the possibility of considering an analytical standard Chapman modeled ionosphere (Chapman, 1931), numerical representations of the ionosphere can also be considered. The corresponding 3-D electron density matrixes (as the one shown in Fig. 1) are formed by cells extending tens of kilometers in latitude and in longitude, and a few kilometers in altitude. At each step of integration the algorithm evaluates the actual cell C_{ijk} along the path, according to the flowchart shown in Fig. 6, and hence takes the corresponding value of electron density, which is considered the same inside the whole cell.

149 Concerning the numerical 3-D representation of the ionosphere, this is based either on the
150 Adaptive Ionospheric Profiler (AIP) model developed by Scotto (2009) or on the model proposed
151 by Pezzopane et al. (2011). Both models derive from the International Reference Ionosphere
152 (Bilitza, 2008) and rely on the real-time autoscaling performed both by Autoscala (Pezzopane and
153 Scotto, 2007, 2008, 2010) and by ARTIST (Reinisch and Huang, 1983, Galkin and Reinisch, 2008).
154

155 **5. Consistency check of the integration algorithm**

156 In order to check the error due to the integration algorithm, the time delay of the wave as it results
157 from the RT computation performed by IONORT, and the time delay of the wave propagating along
158 the oblique virtual path at the speed of light c , were compared at different frequencies. In order to
159 calculate the latter time delay, a flat reflector is assumed at an altitude compatible with the vertical
160 virtual height of reflection. The relation between the vertical and oblique frequencies is given by the
161 secant law $f_v = f_{ob} \cos \varphi$ (Davies, 1990), where φ is the incidence angle, f_v is the vertical frequency,
162 and f_{ob} is the oblique frequency. The equality of the two time delays is assured by the Breit-Tuве
163 and Martyn theorems (Davies, 1990) in case of a monotonically increasing electron density profile.
164 Bianchi et al. (2011) ran such a test by employing a numerical electron density matrix, and the
165 corresponding results are shown in Fig. 7. It came out that the IONORT ray tracing algorithm fits
166 nearly perfectly the theory stated by the two aforementioned theorems. This means that the relative
167 error Δt_{error} between the time delay t_{calc} calculated by IONORT and the simulated time delay t_{virt} ,
168 calculated according to the Breit-Tuве and Martyn theorems, is only due to the discrete integration
169 step.

170

171 **6. Conclusions**

172 In this paper, an applicative software tool package running under Windows operating system,
173 named IONORT, capable to solve the ray tracing for HF waves propagating in the ionosphere was
174 described. The integration algorithm of IONORT is coded in Fortran, while the GUI managing the

175 input needed to the integration algorithm, and the corresponding numerical and graphical output, is
176 coded in MATLAB. This GUI facilitates noticeably the numerical input data entry made by the user
177 and at the same time performs a useful 2-D/3-D visualization of the ray path.

178 From a numerical point of view, in order to calculate the coordinates of the ray and the three wave
179 vector components along the path as dependent variables, IONORT solves at least six first order
180 differential equations, the group time being the independent variable of integration.

181 The consistency of the integration algorithm was checked by comparing real and virtual time
182 delays.

183 The possibility offered to the user of choosing among different ionospheric electron density
184 models, having validity in the area of interest, gives IONORT the necessary flexibility. It is worth
185 noting that this last feature makes IONORT a valuable tool to test the goodness of the 3-D electron
186 density representation of the ionosphere calculated by a definite model. In fact, given a radio link
187 for which oblique soundings are routinely carried out, IONORT gives the possibility to generate
188 synthesized oblique ionograms over the same radio link. The comparison between synthesized and
189 measured oblique ionograms, both in terms of the ionogram shape and in terms of the maximum
190 usable frequency characterizing the radio path, offers a great opportunity to understand how well
191 the model can represent the real conditions of the ionosphere (Angling and Khattatov, 2006).
192 Anyhow, this issue will be presented and discussed in a forthcoming paper.

193

194 **Appendix A. Equation (1.1) and (1.2) in spherical coordinates**

195 In spherical coordinates the equations (1.1) and (1.2) become

196

$$197 \quad \frac{dr}{d\tau} = \frac{\partial H}{\partial k_r}, \quad (A.1)$$

198

$$\frac{d\theta}{d\tau} = \frac{1}{r} \frac{\partial H}{\partial k_\theta}, \quad (\text{A.2})$$

$$\frac{d\varphi}{d\tau} = \frac{1}{r \sin \theta} \frac{\partial H}{\partial k_\varphi}, \quad (\text{A.3})$$

$$\frac{dk_r}{d\tau} = -\frac{\partial H}{\partial r} + k_\theta \frac{d\theta}{d\tau} + k_\varphi \sin \theta \frac{d\varphi}{d\tau}, \quad (\text{A.4})$$

$$\frac{dk_\theta}{d\tau} = -\frac{1}{r} \left(-\frac{\partial H}{\partial \theta} - k_\theta \frac{dr}{d\tau} + k_\varphi r \cos \theta \frac{d\varphi}{d\tau} \right), \quad (\text{A.5})$$

$$\frac{dk_\varphi}{d\tau} = -\frac{1}{r \sin \theta} \left(-\frac{\partial H}{\partial \varphi} - k_\varphi \sin \theta \frac{dr}{d\tau} - k_\theta r \cos \theta \frac{d\theta}{d\tau} \right), \quad (\text{A.6})$$

where H is the Hamiltonian, k_r , k_θ , k_φ (see Fig. 2) are the components of the wave vector along r , θ , and φ . The Hamiltonian H is a constant during the ray propagation, and for the IONORT algorithm the following relation was chosen

$$H(r, \theta, \varphi, k_r, k_\theta, k_\varphi) = \frac{1}{2} \text{Re} \left[\frac{c^2}{\omega^2} (k_r^2 + k_\theta^2 + k_\varphi^2) - n^2 \right], \quad (\text{A.7})$$

where n is the phase refractive index, c is the speed of light, and ω is the fixed angular frequency of the wave.

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300 **Fig. 1.** 3-D matrix of the electron density, where i and j vary with the latitude and longitude, and k
301 with the altitude, respectively. Column on the left composed by k cells represents the vertical
302 electron density profile V_{ij} . Ray path from a TX point to a RX point is represented by a dotted
303 curve. Because of the involved distance, the 3-D matrix has a spherical shell shape.

304

305 **Fig. 2.** Geocentric reference system in spherical (r, θ, φ) and Cartesian (x, y, z) coordinates. \mathbf{k} is the
306 wave vector and in red, in blue and in violet the corresponding projections along versors i_r , i_θ , and
307 i_φ .

308

309 **Fig. 3.** Flowchart of IONORT application.

310

311 **Fig. 4.** GUI of IONORT program. “Main parameters” and “Step” frames are related to the input
312 data. “Model” frame shows the analytical and numerical ionospheric models that can be chosen by
313 the user. “Ray” frame gives the user the possibility to choose between the two different polarization
314 of the wave, ordinary or extraordinary. “Results” frame shows the numerical output values. “RUN”
315 button launches the integration algorithm. “Reset” button clears all the different outputs. At the
316 bottom and on the right side, the 2-D and the 3-D visualizations of the ray path are respectively
317 shown by considering a TX point at 43.06°N of latitude and at 10.03°E of longitude, for a fixed
318 frequency equal to 6 MHz and for a 5° elevation-step procedure from 0° to 30° .

319

320 **Fig. 5.** Same as Fig. 3 for a fixed elevation angle equal to 15° and for a 2 MHz frequency-step
321 procedure from 2 MHz to 24 MHz.

322

323 **Fig. 6.** Flowchart of subroutine “cellfind”.

324

325 **Fig. 7.** Time group delays, t_{virt} and t_{calc} , calculated by employing a numerical electron density
326 matrix, and corresponding percentage relative error.

327

328 **Table 1.** First 25 components of the input vector W .

329

330 **Table 2.** Analytical and numerical electron density models that can be used by IONORT. NF and
331 WF stand for no magnetic field and with magnetic field respectively. Because of the ray path does
332 not change significantly at the employed operating frequencies, the models do not include the
333 neutral particles-electrons collision frequency.

Figure 1
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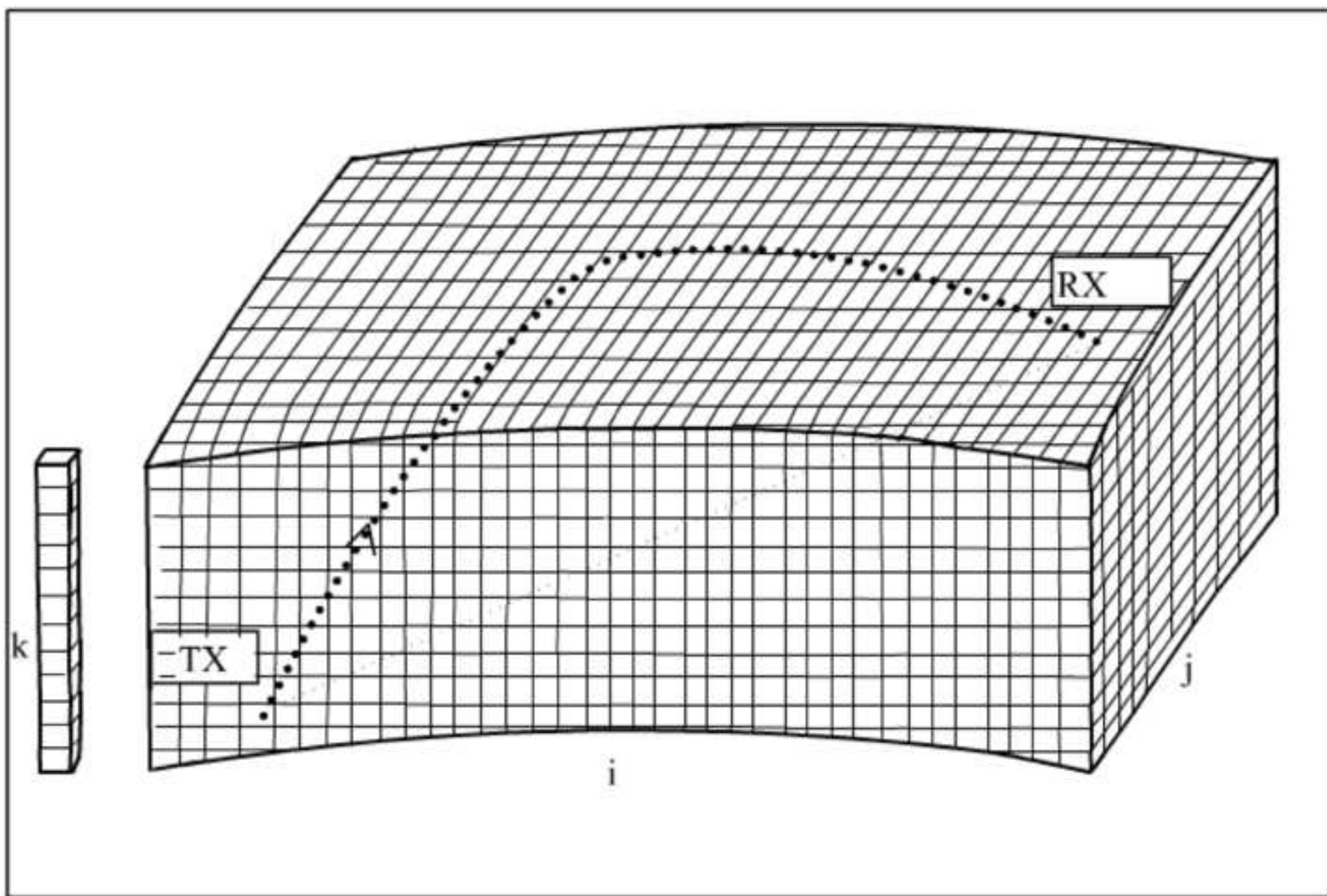


Figure 2
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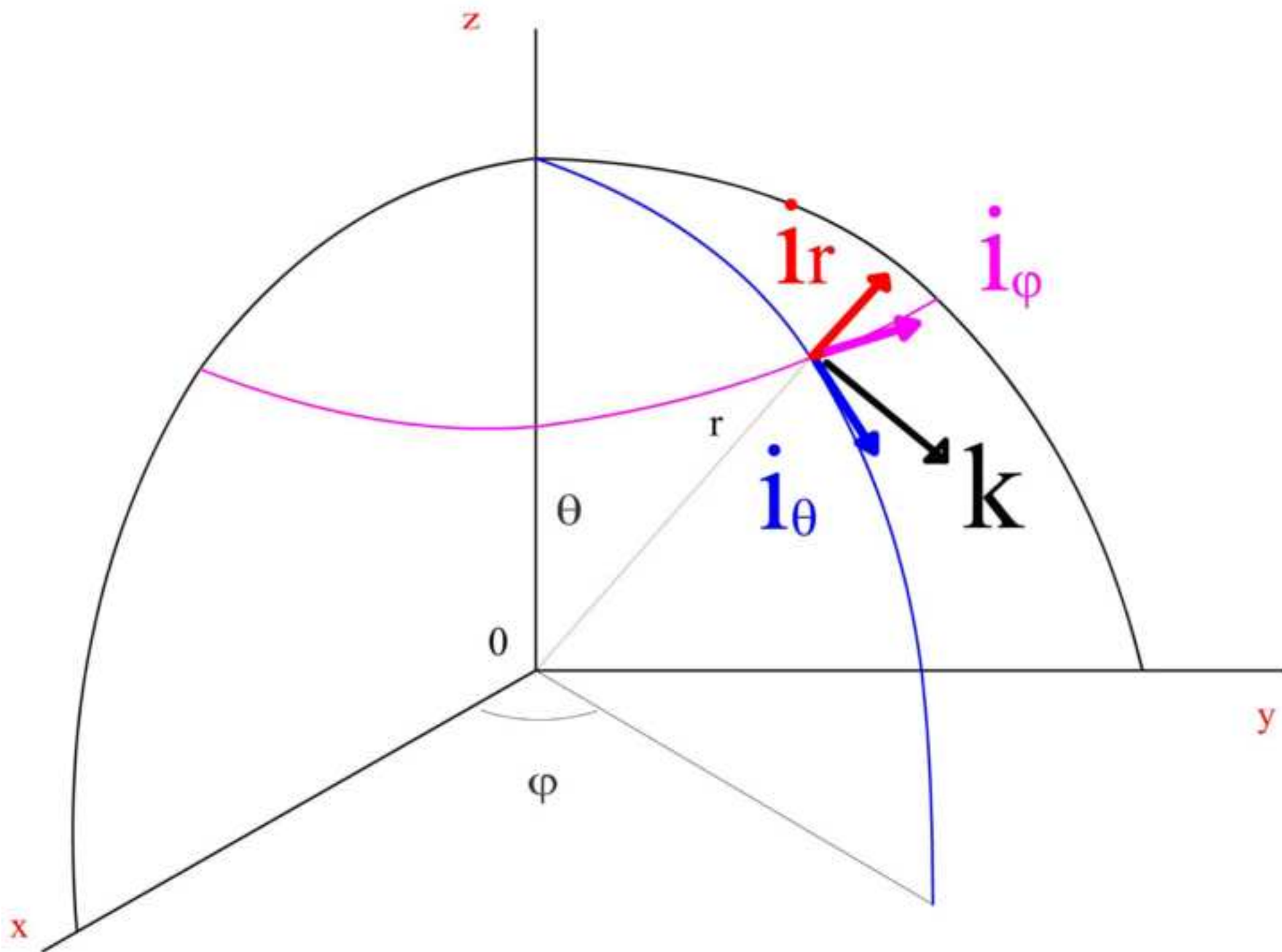


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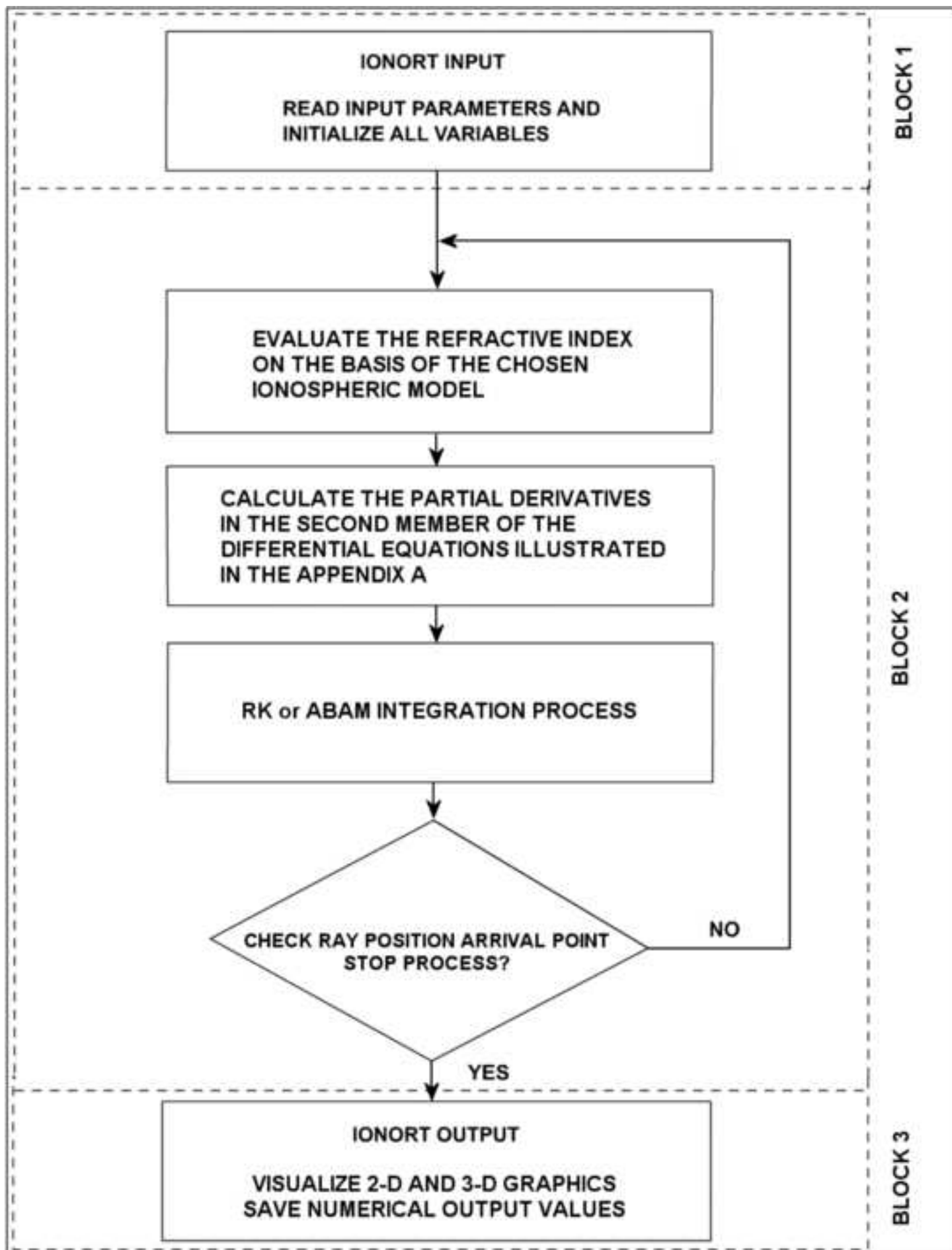


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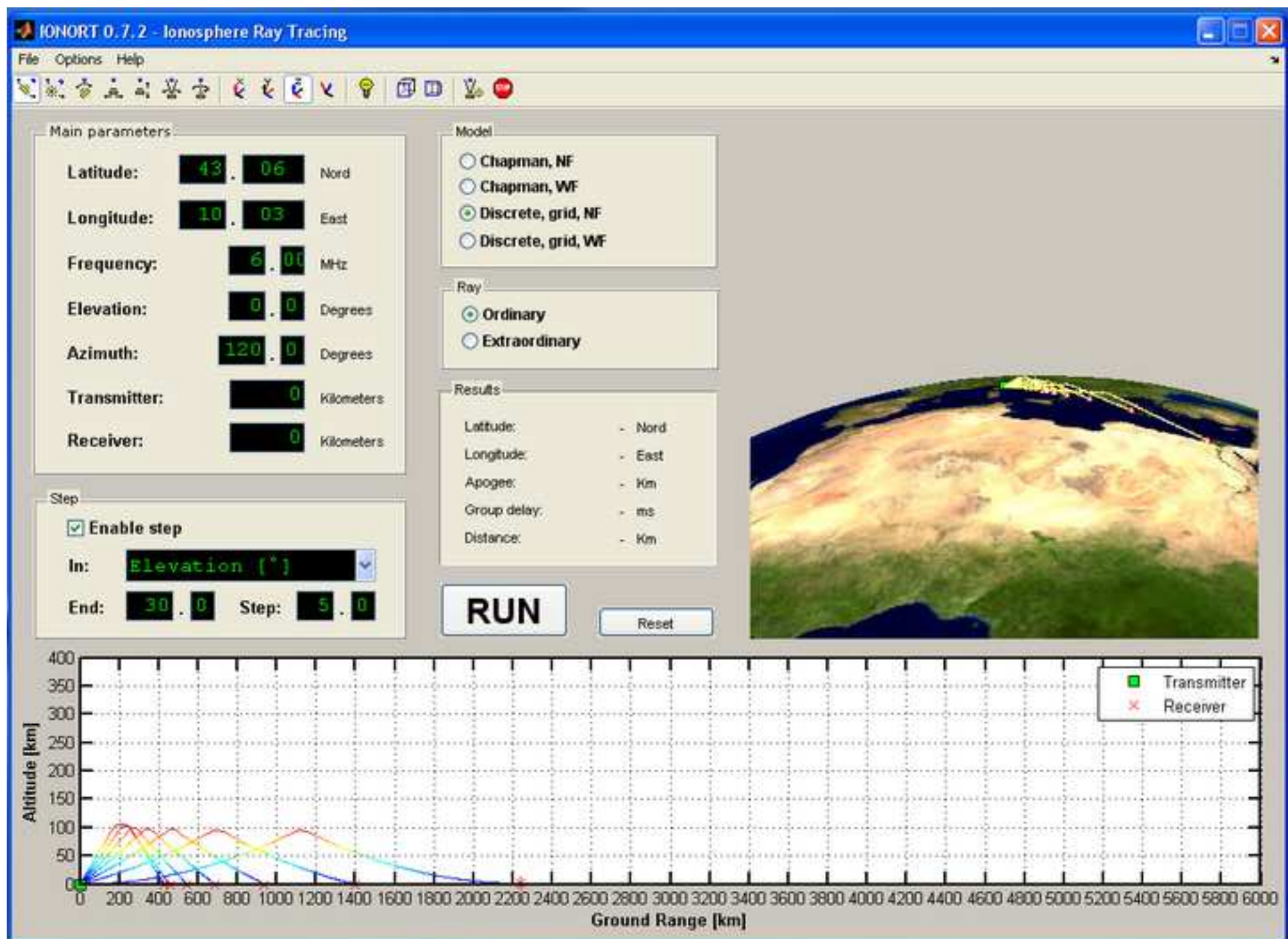


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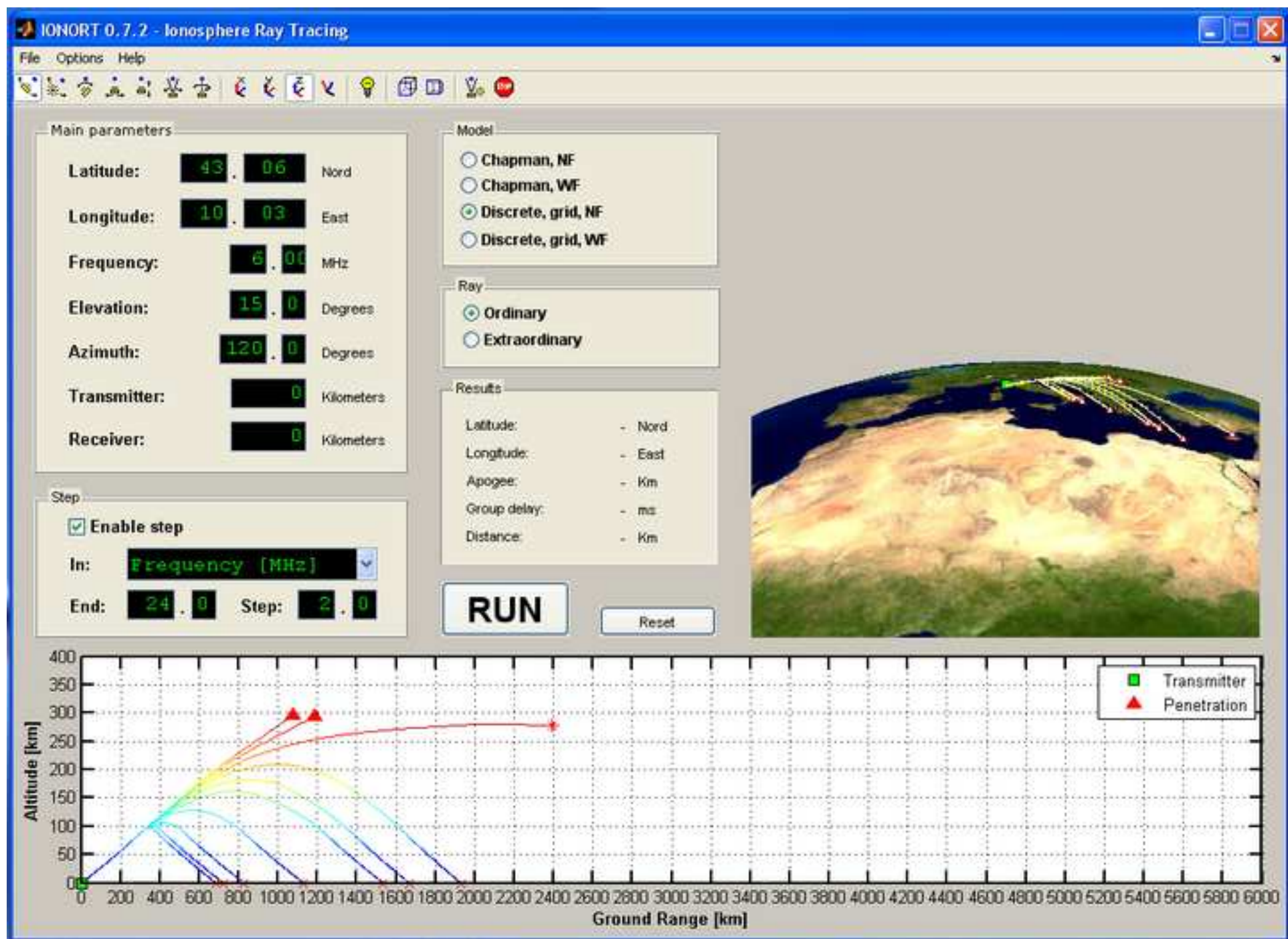


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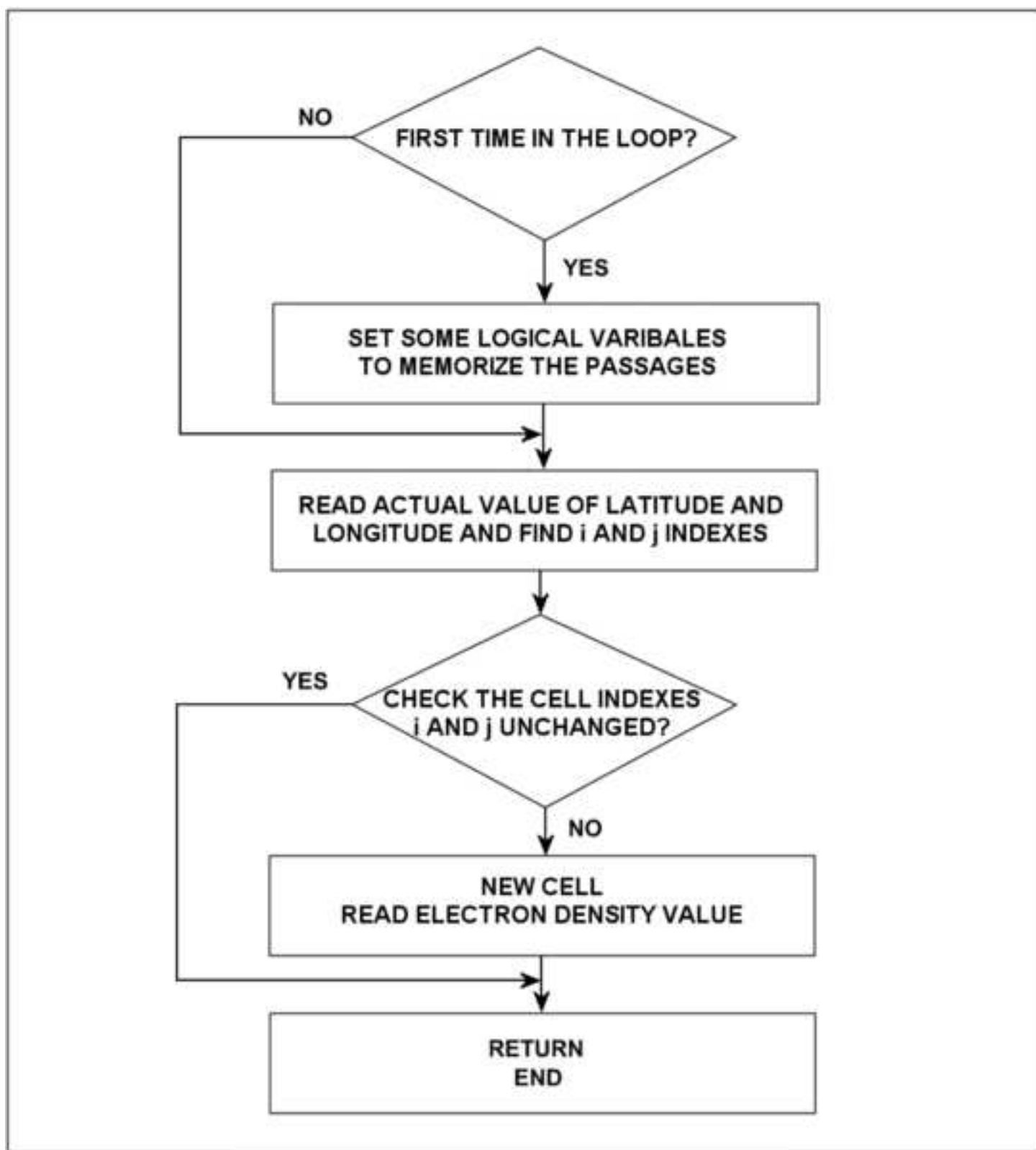


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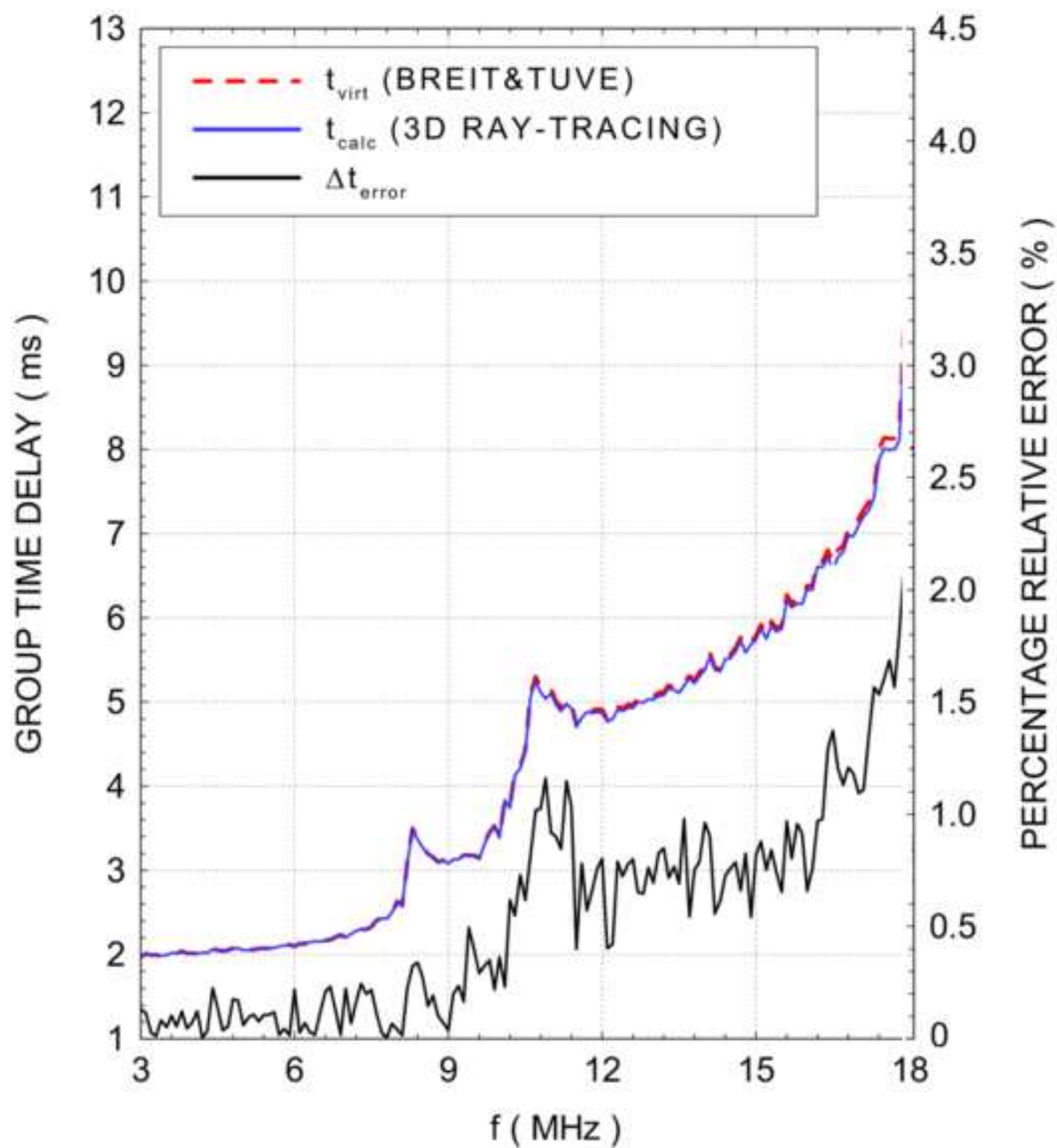


Table 1

W vector component	Parameter	Description	Value
W1	RAY	Radio Wave Mode	1 = Ordinary -1 = Extraordinary
W2	EARTH	Earth Radius	6371 km
W3	XMTRH	Height of TX	km
W4	TLAT	North geographic latitude of TX	rad
W5	TLON	East geographic latitude of TX	rad
W6	F	Frequency	MHz
W7	FBEG	Initial frequency	MHz
W8	FEND	Final frequency	MHz
W9	FSTEP	Frequency step	MHz
W10	AZI	Azimuth angle of transmission	rad
W11	AZBEG	Initial azimuth	rad
W12	AZEND	Final azimuth	rad
W13	AZSTEP	Azimuth step	rad
W14	BETA	Elevation angle of transmission	rad
W15	ELBEG	Initial elevation	rad
W16	ELEND	Final elevation	rad
W17	ELSTEP	Elevation step	rad
W20	RCVRH	Height of RX	km
W21	ONLY	Reflected/not reflected rays	0 = only reflected rays 1 = reflected/penetrating rays
W22	HOP	Maximum number of hops	real
W23	MAXSTP	Maximum number of steps for hops	real
W24	PLAT	North geographic latitude of north geomagnetic pole	rad
W25	PLON	East geographic longitude of north geomagnetic pole	rad

Table 2

Model	Description	Magnetic Field
Chapman, NF	Analytical electron density profiles	No
Chapman, WF	Analytical electron density profiles	Yes
Discrete, grid, NF	Numerical gridded electron density profiles	No
Discrete, grid, WF	Numerical gridded electron density profiles	Yes

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